Remarks on Inflation and Noncommutative Geometry

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We briefly discuss some possible cosmological implications of noncommutative geometry. While the noncommutativity we consider does not affect gravity, it can play an important role in the dynamics of other fields that are present in the early universe. We point out the possibility that noncommutativity may cause inflation induced fluctuations to become non-gaussian and anisotropic, and may modify the short distance dispersion relations.

It has long been recognized that cosmology provides a fertile testing ground for theories beyond the standard model of particle physics. For string theory, in fact, cosmology may one day provide the most accessible way to probe the theory experimentally. In this regard, inflation is an especially promising framework as the enormous growth of scales in the early universe stretches regions on the order of the Planck scale — the likely relevant scale for string theory — to the much larger scales of relevance for cosmology.

Recently, there has been significant interest in noncommutative geometry due to developments in matrix theory [1] and the realization [2–6] that noncommutative spacetime arises naturally in string and M-theory when a certain background gauge field is turned on. In particular, it was shown [4–6] that in the presence of a constant NS $B_{\mu\nu}$-field, the endpoints of the open string obey the commutation relation

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \tag{1}$$

where the $\theta^{\mu\nu}$ are entries of an antisymmetric real constant matrix of dimension length squared. The relation between θ and B can be found in [4–6]. Moreover the commutation relations of the string modes are modified. These relations have been employed directly to construct noncommutative open string theory at any loop order [7]. Note that perturbatively the noncommutativity is only felt by open strings, closed strings are not affected by the B-field.

A number of authors have studied the possible phenomenological effects of such noncommutativity [8]. In this brief note, using basic properties of noncommutative field theory [6,9–11], we point out some possible cosmological signatures. The idea is that if spacetime is indeed noncommutative on short distance scales, this may have an impact on early universe physics. As above, we work in the context of inflation which allows such short scale noncommutativity to amplify into large scale cosmological implications. Specifically, we focus on the generation of density perturbations. In the usual setup, quantum fluctuations of the inflaton field, after suitable tuning of its potential, can give rise to the requisite density perturbations for structure formation. We reexamine these calculations in the noncommutative framework and point out features that can differ from the usual commutative case. Inflationary cosmology in noncommutative geometry from a different perspective was studied in [12].

It is convenient to work in the dual language of fields whose algebraic structure is defined by the Moyal product

$$(f * g)(x) = e^{i\frac{\theta^{\mu\nu}}{2}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\zeta^\nu}} f(x+\xi)g(x+\zeta)|_{\xi=\zeta=0}, \tag{2}$$

which is associative, noncommutative and satisfies

$$\int f * g = \int g * f = \int fg. \tag{3}$$

Using this $*$-product, field theory on a noncommutative space (ie. $\theta^{\mu\nu} \neq 0$ only for $\mu, \nu \neq 0$) can be easily formulated. Since it is not clear how to quantize a theory with nonzero θ^{0i} [13,14], we will restrict ourselves to spatial noncommutativity. Realizing a noncommutative field theory simply amounts to replacing the usual multiplication of functions by the $*$-product. For example, noncommutative QED is given by

$$S = -\frac{1}{4}\int d^4x (F_{\mu\nu} * F^{\mu\nu} + i\bar{\psi} * \not{D} * \psi) \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]_*$ and $D_\mu\psi = \partial_\mu\psi + igA_\mu * \psi$ for a Dirac spinor. Note that due to (3), the quadratic part of the action is not modified by θ. The theory is non-local as the $*$-product gives rise to an infinite number of derivatives in the action.

Before we turn to inflation, we remark that when quantum effects are taken into account, the naive $\theta \to 0$ limit may not be smooth in the sense that in this limit a noncommutative field theory does not always reduce to its commutative $\theta = 0$ counterpart [10,15–17]. For example, the beta function for noncommutative QED is found to be [15,16,10]

$$\beta = -\frac{g^3}{16\pi^2}\left(\frac{22}{3} - \frac{4}{3}N_f\right), \tag{5}$$

where N_f is the number of Dirac fermions. Note that the new (negative) term is due to the self-interaction of the noncommutative photons and is independent of θ so long as θ is nonzero. Summing together the contributions from the standard model matter fields, one finds that the beta function is negative. This is in conflict with our expectation that the $U(1)$ coupling is not asymptotically free. Moreover once θ is turned on, gauge invariance and the fact that some standard model fields are charged under both $U(1)$ and $SU(2) \times SU(3)$ imply that the noncommutative gauge symmetry has to be enlarged to $U(1) \times U(2) \times U(3)$ [6]. With $U(3)$ as the color group, the existence of baryons becomes a problem. However by embedding the noncommutative theory in string theory, one may be able to resolve these issues with the addition of new degrees of freedom [10] which become effective at the scale $1/\sqrt{\theta}$. More work has to be done to substantiate this picture. These new degrees of freedom have implications for the signatures studied in [8].

The above considerations suggest an alternative appealing framework in which conventional commutative geometry emerges from a fundamental noncommutative starting point: the degree of noncommutativity may be scale-dependent (or temperature-dependent with Λ replaced by T below). For example,

$$\theta^{\mu\nu} = \begin{cases} \theta^{\mu\nu}, & \text{if } \Lambda > \Lambda_0, \\ 0, & \text{if } \Lambda < \Lambda_0. \end{cases} \tag{6}$$

We note that (6) is not the same as a spatially varying θ and that a scale or temperature dependent θ is consistent with associativity. To our knowledge, this simple possibility has not been discussed before. The scale Λ_0 can be interpreted as the Wilsonian cutoff of the field theory. As long as Λ_0 is much higher than the electroweak or SUSY breaking scale, the problems mentioned above can be avoided. An interesting scenario is to suppose that Λ_0 is significantly larger than the electroweak scale, but smaller than the scale of inflation (which is roughly the GUT scale if the inflaton is embedded in a GUT model, or the Planck scale in models of chaotic inflation) so that one has a noncommutative universe during the inflationary period *. As we now discuss, since the dynamics of the inflaton field in such a scenario is governed by a noncommutative field theory which is non-local and Lorentz non-invariant, the density perturbations due to quantum fluctuations of the inflaton field are different from that found in usual inflation.

*Recently, an interesting scenario in which the commutative world is recovered in the low energy regime, together with a decoupling of the above mentioned problematic $U(1)$ degree of freedom and with supersymmetry broken dynamically is discussed in [18].

One of the central ideas of modern cosmology is that the observed inhomogeneity of the universe has its origin in the quantum fluctuations of fields that are present during inflation [19,20]. These quantum fluctuations, generated during the "slow rolling" period were initially taken outside the horizon by the rapid inflation and their form was frozen until they re-entered the horizon. These primordial perturbations then grew with time due to gravitational instability and eventually became the observed classical structures of the universe.

The precise form of these fluctuations depends on the kinematics and dynamics of the inflaton field. For example, in the simplest inflationary models, the quantum fluctuations have a gaussian distribution (for a review of inflationary cosmology, see for example [21]), with amplitudes governed by free field dynamics. But non-gaussian perturbations are possible in more complicated models. (For example, higher derivative inflationary dynamics was considered in [22]. Interestingly, the interactions in a noncommutative field theory generically contain higher derivative terms of the kind in [22].) Here we note that even in free noncommutative field theory, the kinematics are such that a non-gaussian distribution is naturally obtained. The deviation from gaussian processes is determined by the magnitude of the noncommutativity parameter θ. We also note that the dynamics of noncommutative field theory can lead to yet other deviations from traditional density perturbation calculations.

For simplicity, we assume that the noncommutativity of the universe at the inflation scale takes the form of (2). Since we need to consider products of fields at different points, we also need the more general $*$-product [11]

$$f(x_1) * g(x_2) = e^{i \frac{\theta^{\mu\nu}}{2} \frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2}} f(x_1) g(x_2), \tag{7}$$

This is easily shown to be associative.

Now we want to study quantum fluctuations of the inflaton ϕ. During the slow roll period of inflation, the potential energy $V(\phi)$ is approximately constant and the universe can be described by the de Sitter spacetime

$$ds^2 = dt^2 - e^{2Ht} d\mathbf{x}^2. \tag{8}$$

We will assume that the inflaton field comes from the open string sector, even though *a priori*, it can be a closed string state. However, this assumption is quite natural in brane-world inflationary scenarios, (see *e.g.*, [23]). The action for the inflaton ϕ is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi * \partial_\nu \phi - V_*(\phi) \right). \tag{9}$$

Here we take gravity as a background that is not affected by the noncommutativity. More general considerations with gravity also seeing the noncommutativity can be found in [24]. Since $\theta^{0i} = 0$ and the metric is independent of $\mathbf{x}$, (3) can be generalized with an arbitrary time-dependent factor inserted. Note that the $*$-product in (9)

is taken to be defined with respected to the comoving coordinates $\mathbf{x}$. That is to say we are considering a sceranio in which noncommutativity parameter $\theta^{\mu\nu}$ is constant in the comoving frame. In physical coordinates this means that θ is growing with the scale factor, something that seems reasonable since the B-field is the superpartner of the metric. If θ subsequently drops to zero, say at the end of inflation, this should yield a viable cosmology †. One obtains the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\Delta\phi + \delta V_*/\delta\phi = 0, \tag{10}$$

where Δ is the usual 3-dimensional Laplacian. Until a few Hubble times after the horizon exit, one can drop the V_*'' term [20]. In more complicated models, effects of the potential will have to be taken into account. We will take ϕ to be free, except for some general comments at the end. Even in this case, we will show that noncommutativity can yield deviations from the usual gaussian density perturbations. This is purely a result of the "background" noncommutative geometry. The equation for the fluctuations thus takes the free form

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\Delta\phi = 0. \tag{11}$$

Here ϕ represents the fluctuating part of the inflaton; it has the Fourier expansion ($k = |\mathbf{k}|$),

$$\phi(\mathbf{x}, t) = \int_{\mathbf{k}} \frac{1}{\sqrt{2k}}(a_{\mathbf{k}}\psi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.), \tag{12}$$

where $\mathbf{k}$ is the wave vector in the comoving frame, $\psi_{\mathbf{k}}(t) = \frac{iH}{k}(1 + \frac{k}{iH}e^{-Ht})\exp(\frac{ik}{H}e^{-Ht})$, $\int_{\mathbf{k}} \equiv \int \frac{d^3\mathbf{k}}{(2\pi)^3}$ and $\psi_{\mathbf{k}}(t) \sim e^{-ikt}$ for $k/H \gg 1$. Canonical quantization of ϕ imposes the commutation relations

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}'), \quad [a_{\mathbf{k}}, a_{\mathbf{k}'}] = 0. \tag{13}$$

Since ϕ feels the noncommutativity, the relevant n-point correlation function is

$$I_n(\mathbf{x}_1, \cdots, \mathbf{x}_n) = \langle 0|\phi(\mathbf{x}_1, t) * \cdots * \phi(\mathbf{x}_n, t)|0\rangle, \tag{14}$$

where the time dependence is understood. Essentially as in [9] one obtains

$$I_2 = \int_{\mathbf{k}} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{2k}\eta_k(t), \quad \eta_k(t) \equiv e^{-2Ht} + H^2/k^2 \tag{15}$$

which is independent of θ and takes the usual form. As for the 4-point function, it is

$$I_4 = I_2(\mathbf{x}_1 - \mathbf{x}_2)I_2(\mathbf{x}_3 - \mathbf{x}_4) + I_2(\mathbf{x}_1 - \mathbf{x}_4)I_2(\mathbf{x}_2 - \mathbf{x}_3) + \\ + \int_{\mathbf{k}}\int_{\mathbf{k}'} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_3)}}{2k}\eta_k(t)\frac{e^{i\mathbf{k}'\cdot(\mathbf{x}_2-\mathbf{x}_4)}}{2k'}\eta_{k'}(t)e^{-i\mathbf{k}\wedge\mathbf{k}'}, \tag{16}$$

where $k \wedge k' \equiv k_\mu\theta^{\mu\nu}k'_\nu$, with $I_3 = 0$ and more generally $I_{2n+1} = 0$. Note that although ϕ is a free field, the n-point functions depend on θ because of the $*$-product. Note also that because of these contributions, even in free field theory I_4 and generally I_n cannot be factorized in terms of products of I_2. A few Hubble times after horizon exit, the 4-point function in momentum space is

$$I_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = (4\pi^3H^2k_1^{-3}) \cdot \\ [k_3^{-3}\delta(\mathbf{k}_1 + \mathbf{k}_2)\delta(\mathbf{k}_3 + \mathbf{k}_4) + k_2^{-3}\delta(\mathbf{k}_1 + \mathbf{k}_4)\delta(\mathbf{k}_2 + \mathbf{k}_3) \\ + k_2^{-3}\delta(\mathbf{k}_1 + \mathbf{k}_3)\delta(\mathbf{k}_2 + \mathbf{k}_4)e^{-i\mathbf{k}_1\wedge\mathbf{k}_2}]. \tag{17}$$

These quantum fluctuations are carried outside the horizon and lead to curvature fluctuations. By the time these fluctuations re-enter the horizon, the relevant physical processes occur at a lower scale and hence are described by a commutative dynamics. Therefore, they will just set the initial conditions for the subsequent evolution of the perturbations and show up as the observed inhomogeneities of the universe; see for example [25] for a comprehensive treatment. Since I_2 is unmodified, we get the usual power spectrum $P(k) = k^{n-1}$, with spectral index $n = 1$ for the free case. However, since I_4 does not factorize into products of I_2, the subsequent distribution will not be gaussian. We note that this violation of gaussian statistics is independent of the couplings and is universal; in any specific scenario there may be additional model-dependent violations. The non-gaussian fluctuations will be reflected, for example, in the galaxy distribution and the CMB measurements; stronger constraints are expected to come from the latter. By extracting the 4-point function from the existing 4-year DMR maps or more refined sky maps from future experiments such as MAP and PLANCK, one should be able to set a bound on the degree of noncommutativity during inflation.

In the above, we considered the case in which θ is constant in the comoving frame. As we mentioned, one may also consider the case in which θ is constant in the physical frame. All of the above formula are basically unchanged, except that we just have to use a time varying $\theta^{\mu\nu}(t)$,

$$\theta^{\mu\nu}(t) = e^{-2Ht}\theta_0^{\mu\nu}. \tag{18}$$

Due to the exponential decay factor, the effect of noncommutativity gets redshifted away by inflation ‡. However so long as θ is constant in the comoving frame and

† A different scenario in which $\theta^{\mu\nu}$ is constant in the physical coordinates can also be considered. The difference between the two is the cosmological scaling factor. This will have an important difference in the observational predications of the model. We will comment on this later.

‡We thank Will Kinney for important discussions on this point.

shuts down by the end of inflation, this can lead to a small amount of non-gaussianity. Again, we remark that a comoving constant θ is suggested in string theory since the NSNS B-field and the metric g are coupled to each other through their equation of motion. It is thus natural to assume that θ grows since the metric may grow by inflation.

Beyond the universal effects due to noncommutative geometry discussed so far, there will be additional effects arising from the dynamical details of any particular model. One expects that noncommutative interactions will make a θ dependent contribution to the spectrum of fluctuations (similar to the analysis in [26]). In the commutative case, interaction effects are often too small to be observed, so it is worth determining if there are noncommutative models in which their impact is amplified. We also note that nonzero θ may potentially be relevant for understanding the CMB dipole anisotropy. The CMB dipole from DMR has an amplitude 3.358 ± 0.024 mK in a particular known direction [27]. The conventional interpretation invokes the Doppler effect arising from the motion of the solar system with respect to the CMB rest frame. There is room, however, for other possible contributions to the dipole anisotropy. For instance, nonzero θ introduces a degree of anisotropy whose contribution will depend in detail on the form of the interactions coded in V_*. Whether this yields a viable contribution to the dipole is thus a model dependent question that we leave to future work.

Frameworks for studying related issues have been developed in [28] and [29], in which modifications to conventional physics at sub-Planck scales are modeled and their effects on inflation are studied. In [29], the focus is on a quantized spacetime [30] and the string uncertainty relations [31]. In [28], the authors study the effects on inflation due to so called *trans-Planckian* dispersion relations, which have been postulated [32,33] to encode strong gravity effects at sub-Planck scales. We note that even in the absence of gravity, the dispersion relations of a noncommutative quantum field theory are also modified by loop effects [10,34,35]. Since gravity will generally not just modify the propagator, but will also introduce new interactions into the theory, the trans-Planckian dispersion relations are expected to be further corrected. One should then study the combined effects of both on the short distance dispersion relations and determine the impact on the primordial spectrum of perturbations. We leave this interesting analysis for future work.

In this letter we focused mainly on the case of a scale dependent θ. A related scenario is that the world is commutative at low temperature but becomes more and more noncommutative (and non-local) once the temperature is higher than a certain threshold temperature T_0. In string theory, noncommutativity arises when a non-zero background B-field is turned on. In perturbative string theory, this B-field is a modulus, and so its value is arbitrary. However, in four dimensions, the NS B-field is dual to a scalar. And just like the dilaton, it is possible that a potential can be generated for B non-perturbatively. For example, if an isotropic potential of the following inverse symmetry breaking form (see e.g. [36])

$$V(X) = \frac{\mu}{2} \left[1 - \left(\frac{T}{T_0}\right)^2\right] X^2 + \frac{\lambda}{4} X^4 \qquad (19)$$

is generated for $X^2 = \mathbf{B^2}$ and μ, $\lambda > 0$, then for $T < T_0$, the minimum of the potential is at $X = 0$, and spacetime is commutative. For $T > T_0$, $X = 0$ is a maximum and the true minimum occurs at $X^2 = \frac{\mu}{\lambda}((T/T_0)^2 - 1)$. As a result, spacetime becomes noncommutative and rotational invariance is broken at temperature $T > T_0$. Moreover the degree of noncommutativity depends on T. A more thorough understanding of how this model is embedded in string theory and how Lorentz symmetry is restored in the low energy limit would be desirable. We expect that a temperature dependent θ will have interesting consequences on the thermal history of a hot big bang universe.

While we do not pursue it in this paper, our setup can be embedded naturally in the brane world scenario [37–42], if our four-dimensional world is localized on a brane whose worldvolume has a non-zero background B-field in the early universe. The cosmological implications of this scenario have been studied in some detail (see, *e.g.*, [23,43]). Here we expect additional new features as the universe undergoes a period of noncommutativity.

Finally, we remark that noncommutativity may also appear in the extra dimensions [44,45] in which case θ can be taken to be scale independent. In the brane world scenario, this extra-dimensional noncommutativity arises when a higher dimensional brane is wrapped around the extra compactified directions in the presence of a constant B-field. With enough supersymmetry, the universe is effectively four-dimensional and commutative at energies below the threshold of the Kaluza-Klein modes, and θ-modifications are possible only through loops. At energies higher than the Kaluza-Klein threshold, noncommutativity becomes effective. This clearly has implications for collider experiments as well as for the early universe. In the latter case, one has to study the implications for the quantum fluctuations of the higher dimensional noncommutative inflaton. Many interesting questions about this scenario await to be explored. We plan to address some of these issues in future works.

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